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SHORT REPORT

The effect of the intermediate principal stress on pillar strength

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Abstract
Room and pillar mining is an underground mining method that utilizes natural pillar support to control rock mass behavior, ensuring mine stability and a safe mine environment. This study specifically documents the influence of the intermediate principal stress component on the pillar behavior. So far only classical failure criteria ignoring the influence of the intermediate principal stress component were used for underground pillar design. By using an extended Hoek–Brown failure criterion in comparison with the classical Hoek–Brown failure criterion, the influence of the intermediate principal stress component is documented by indicating those areas where the failure criterion is violated. This study demonstrates, that depending on the rock type, the intermediate principal stress component can have a significant effect. Ignoring this influence can lead to uneconomic pillar design and incorrect determination of the factor of safety.

Keywords Hoek–Brown failure criterion · Intermediate principal stress component · Pillar Strength · Room and pillar mining

1 Introduction

Room and pillar mining is a common underground mining method that utilizes natural pillar support to stabilize the underground workings and deformation control. A well-designed pillar system ensures safety of miners and equipment, minimize ore dilution, maximizes productivity, extends mine life, maintains stability of mine structure as well as cost-effective and ecological rehabilitation (Potvin et al. 1990). The most important parameters for room-and-pillar design are depth, orebody strength properties, in-situ stresses, pillar geometry, presence of joint sets or faults, weathering of the rock mass and other prevailing environmental conditions (Sheorey et al. 1986). Empirical methods for pillar design are preferred due to their convenience to apply (Kidega et al. 2022) (Hedley, D.G.F., Grant 1972) (Von Kimmelmann et al. 1984) (Krauland, N., Soder 1987) (Sjoberg 1992) (Potvin et al. 1990) (Lunder, P.J., Pakalnis 1997). However, these empirical pillar design formulas have some inherent simplistic assumptions and weaknesses such as inability to consider complex situations, characterized by various factors such as complex in-situ stresses, existence of faults and fractures or other weak inclusions, irregular shape of ore bodies, topography and complex pillar geometry (Wagner 2003). Numerical simulations are a good alternative for mine design and they are already used on a routine basis using classical failure criteria based solely on maximum and minimum principal stresses, like the Mohr–Coulomb or Hoek–Brown (HB) failure criteria. However, researches have demonstrated at the laboratory scale that the intermediate principal stress effects the failure strength of rocks, for instance:(Kwaśniewski 2013) (Feng et al. 2016),(Mogi 1972),(Feng et al. 2019),(Feng et al. 2020) (Zhang et al. 2013) (Jiang and Zhao 2015). The extended and classical Hoek Brown criteria can estimate the strength of intact rock as well as jointed rock masses by reduction of strength parameters on the basis of rock mass classifications or engineering geological data to suit the rock mass behavior under consideration, like demonstrate for instance by (Li et al. 2021) (Hoek and Brown 2019) (Hoek et al. 2002). At the field scale, geotechnical engineering focus on pillar design under the assumption that the intermediate principal stress component, has insignificant contribution to the strength (C.D Martin, W.G Maybee 2000)

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(Esterhuizen 2007) (Maleki 2017). However, like shown recently by (Chen et al. 2023) for slope stability analysis, the intermediate principle stress component can have some influence on the stability and failure behavior.

This article documents exemplary that the intermediate principal stress component can have a significant influence on the strength and consequently on the load bearing capacity of mine pillars.

2 Extended three-dimensional Hoek–Brown failure criterion

The failure criterion proposed by (Li et al. 2021), which is an extended version of the classical HB-criterion considering the intermediate principal stress component, was selected because it can be expressed analytically by two equations if the stress space is subdived in two parts as shown in Fig. 1 and given by Eqs. (1) and (2).

$$\sigma_1 = \frac{b\sigma_2 + \sigma_3}{b+1} + \sigma_c\left(\frac{m_b}{\sigma_c}\frac{b\sigma_2 + \sigma_3}{b+1} + s\right)^a (\sigma_2 \leq \sigma_2^*) \quad (1)$$

$$\frac{b\sigma_2 + \sigma_1}{b+1} = \sigma_3 + \sigma_c\left(m_b\frac{\sigma_3}{\sigma_c} + s\right)^a (\sigma_2 \geq \sigma_2^*) \quad (2)$$

where σ_1, σ_2 and σ_3 are the major,intermediate and minor principal stress respectively. σ_c is the unconfined compressive strength. m_b, s and a are rock mass material constant.

All the parameters of the extended HB-criterion can be determined from uniaxial and triaxial tests. Laboratory testing has documented that peak triaxial strength can be increased by up to about 20% for certain types of rock and optimum values of σ_2 (Haimson and Chang 2000) (Hu et al. 2022) (Feng et al. 2016) (Walton 2021) (Mogi 1971) (Ingraham et al. 2013). Equation (1) shows that for low σ_2, σ_3 of the classical HB-criterion can be replaced by $\frac{b\sigma_2+\sigma_3}{b+1}$ and Eq. (2) shows that for high σ_2, σ_1 of the classical HB-criterion can be replaced by $\frac{b\sigma_2+\sigma_1}{b+1}$. The value b controls the shape of the failure envelope. The intermediate principal stress component increases the strength and a value of $b=0$ (ignoring the intermediate principal stress component) signifies the lower bound of the strength envelop or with other words, the classical HB-criterion is always conservative. A simple test of Eqs. (1) and (2) can be used to decide which equation should be applied for the actual stress state: the equation giving the lower values for σ_1 has to be used for the extended HB-criterion (see also flowchart in Fig. 3).

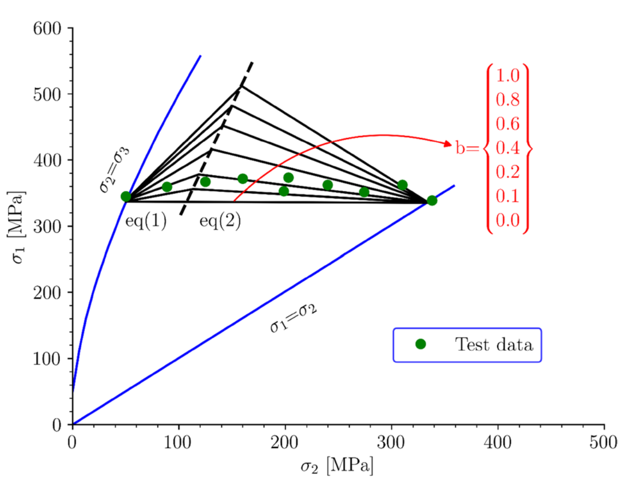

Fig. 1 Extended HB-criterion for different values of parameter b (Li et al. 2021)

3 Numerical analysis

3.1 Numerical model set-up and calculation procedure

To investigate the potential effect of the intermediate principal stress component in general, a 3-dimensional numerical pillar model was set-up on the basis of the explicit finite difference code FLAC^{3D}. In this study, we consider a pillar situated in a large field of pillars (regular room and pillar mining scheme) under assumption of quasi-isotropic and quasi-homogeneous material behavior, which allows to reduce the model to a ¼-model. The model set-up is illustrated in Fig. 2 representing a ¼-model of a pillar with adjacent room.

The outer model dimensions are 8 m × 7 m × 20 m. The pillar height is 5 m, and width and length are of the pillar are 3.5 m and 4 m, respectively. The virgin stress state is characterized by a vertical stress component of 25 MPa and horizontal stresses of 12.5 MPa. At model bottom and vertical

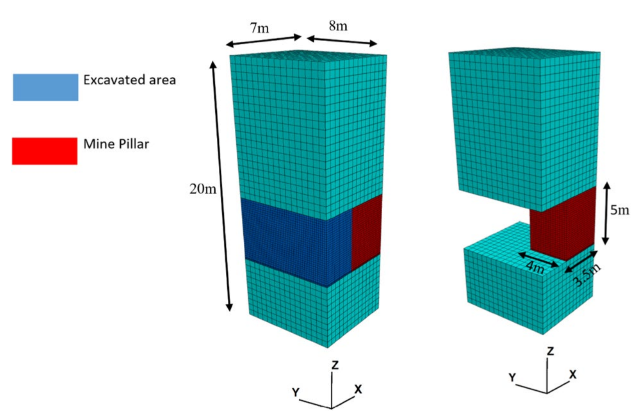

Fig. 2 Numerical model set-up (left: virgin rock mass, right: after excavation)

outer boundaries, the normal displacements are fixed. At the top of the model, a vertical pressure of 25 MPa is applied. The zones are densely populated at the area of excavation to increase accuracy, with each zone occupying a volume of 0.0047 m^3. In the floor and roof regions, each zone occupies 0.125 m^3. There are 67,200 zones in the model before excavation with total volume of 910 m^3. The material parameters for the classical HB-model used for this exemplary study are given in Table 1.

Stress-strength analysis under pure elastic conditions is conducted to evaluate the violation of failure envelope by verifying the actual stress state violates the yield surface of the classical and extended HB-criteria. The purpose is to identify zones of potential failure around the mine pillar. For this evaluation the parameters k_1, k_2 and k_3 are determined for each zone. Equations 3 and 4 specify the extended HB-criterion and Eq. (5) the classical HB-criterion adopted from (Li et al. 2021). The flowchart shown in Fig. 3 is used to determine the stability numbers k_1, k_2 and k_3 to identify zones, where the failure criterion is violated for the classical as well as the extended HB-criterion. k_1, is valid for the region with lower intermediate principal stress ($\sigma_2 \leq \sigma_2^*$) and k_2 is valid for the region with higher intermediate principal stress ($\sigma_2 \geq \sigma_2^*$) according to Fig. 1.

$$\sigma_1 - \left[\frac{b\sigma_2 + \sigma_3}{b+1} + \sigma_c\left(\frac{m_b}{\sigma_c}\frac{b\sigma_2 + \sigma_3}{b+1} + s\right)^a\right] = k_1 \quad (3)$$

Table 1 Mechanical properties of the Chelungpu siltstone I (Li et al. 2021) for the classical HB-criterion

Density (kg/m^3)	Shear Modulus (GPa)	Bulk Modulus (GPa)	a	s	Parameter (mi)	Unconfined compressive strength (MPa)
2500	60	40	0.5	1	13.91	77

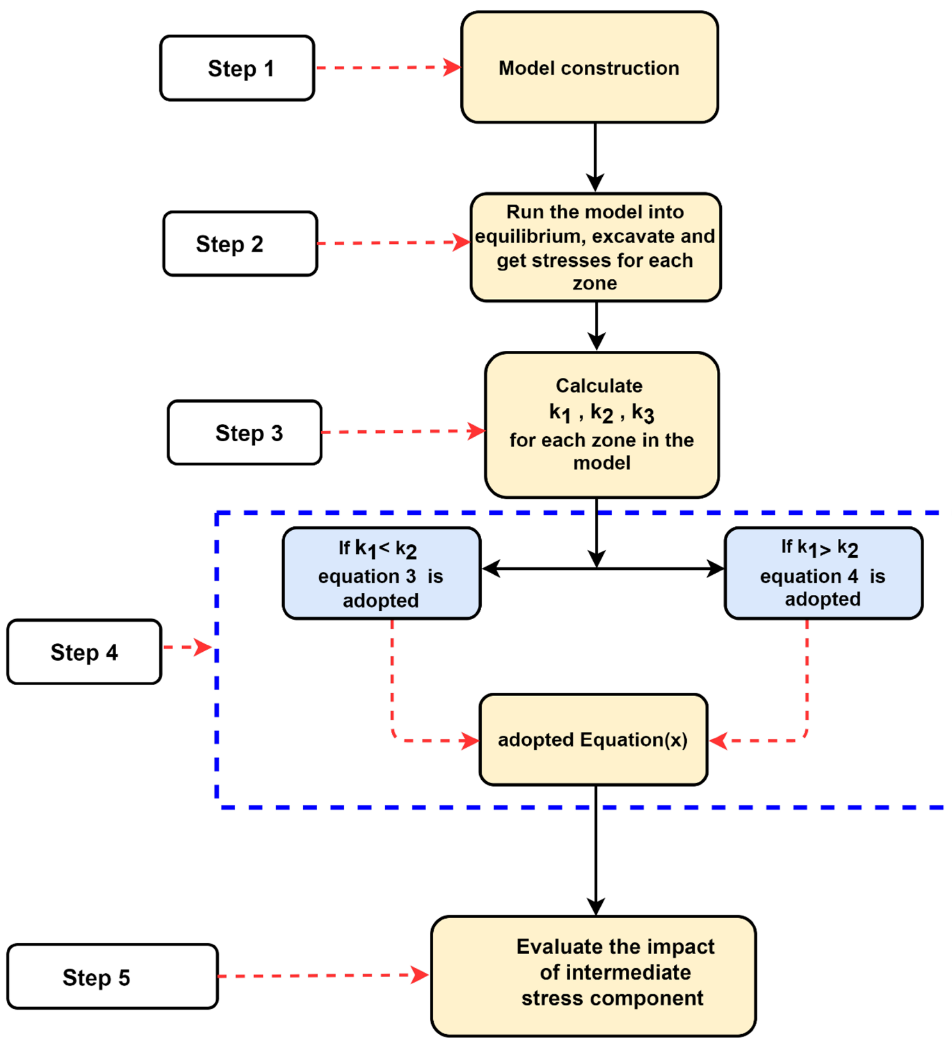

Fig. 3 Flowchart to detect zones violating the failure envelope

$$\sigma_1 - \left[\left((b+1)\left(\sigma_3 + \sigma_c\left(m_b\frac{\sigma_3}{\sigma_c} + s\right)^a\right)\right) - b\sigma_2\right] = k_2 \quad (4)$$

$$\sigma_1 - \left[\sigma_3 + \sigma_c\left(m_b\frac{\sigma_3}{\sigma_c} + s\right)^a\right] = k_3 \quad (5)$$

4 Simulation results

K-values equal or greater than 1 indicate a violation of the strength criterion. For b-value of zero the classical HB-criterion coincides with the extended HB-criterion as illustrated by Fig. 4. Therefore, region of strength exceedance is 24.21m^3 for all three cases (k_1, k_2 and k_3).

The selected rock (Chelungpu siltstone I) according to Li et al. (Li et al. 2021) has a b-value of 0.464. Figure 5 shows the simulation results for the k-values. Strength violation according to k_1 is concentrated at the middle of the pillar height, whereas k_2 is concentrated at the transition from the pillar to the roof and floor area. Considering the realistic b-value of 0.464, the volume of strength exceedance (8.02 and 6.09 m^3, respectively, in total 14.11 m^3, see Figs. 5a and b) is much smaller than predicted by the classical HB-criterion with 24.21 m^3.

The Fig. 6a and b shows the results when parameter b is zero (ignoring the influence of the intermediate principal stress) and 0.464, respectively. The volume of plastic zones indicated by pink colored zones (> 1.0) are 24.21m^3 and 14.11m^3 for classical and extended Hoek Brown criteria, respectively. This implies that the intermediate principal stress component has strengthening effect on pillar stability.

Figure 7 illustrates the effect of different b-values on the extent of the area with strength violation. According to (Li et al. 2021) realistic b-values can range from about 0.2 to nearly 1.0 depending on the type of rock. Table 2 shows the total volume of zones with strength exceedance by applying different b-values.

5 Conclusions

Although laboratory tests have documented the influence of the intermediate principal stress component, this knowledge is yet to be fully incorporated and integrated into the engineering practice, where still strength criteria ignoring the influence of the intermediate principal stress component like the classical Mohr–Coulomb or the classical HB-criterion are dominating. Applying these classical approaches is always conservative in terms of safety and stability considerations. However, consideration of the intermediate principal stress component leads to a

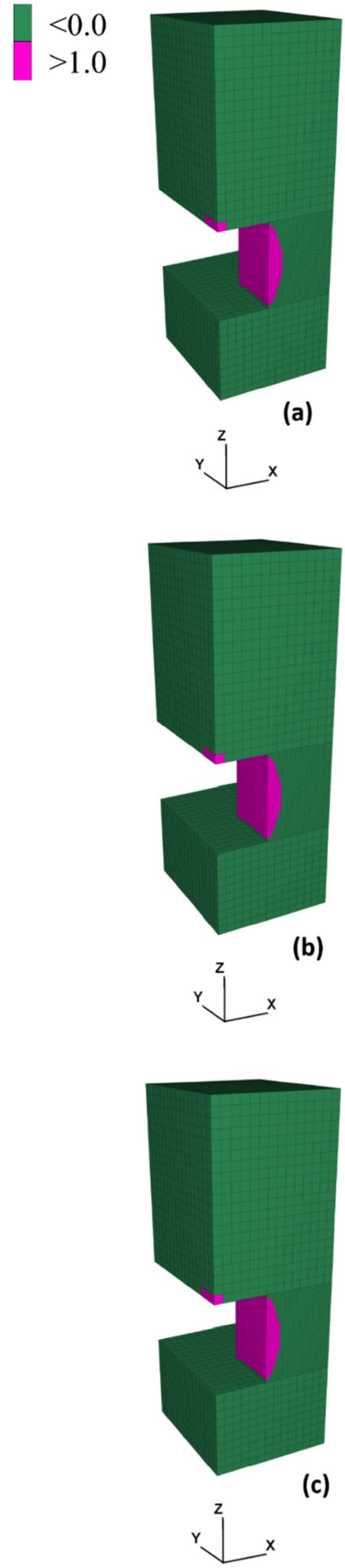

Fig. 4 Values **a** k_1 and **b** k_2 for the extended HB-criterion with b-value of zero and **c** k_3 for the classical HB-criterion

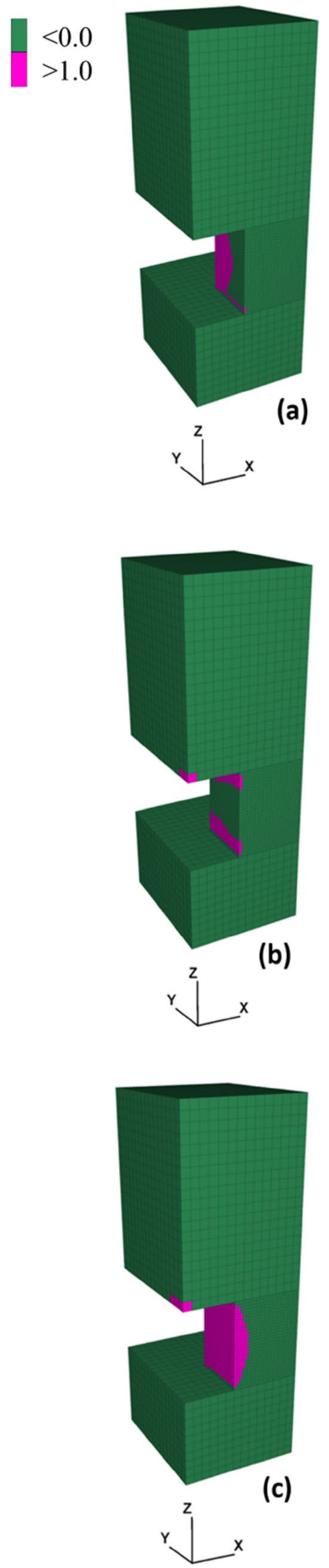

Fig. 5 Values **a** k_1 and **b** k_2 considering a b-value of 0.464 and **c** for the classical HB-criterion

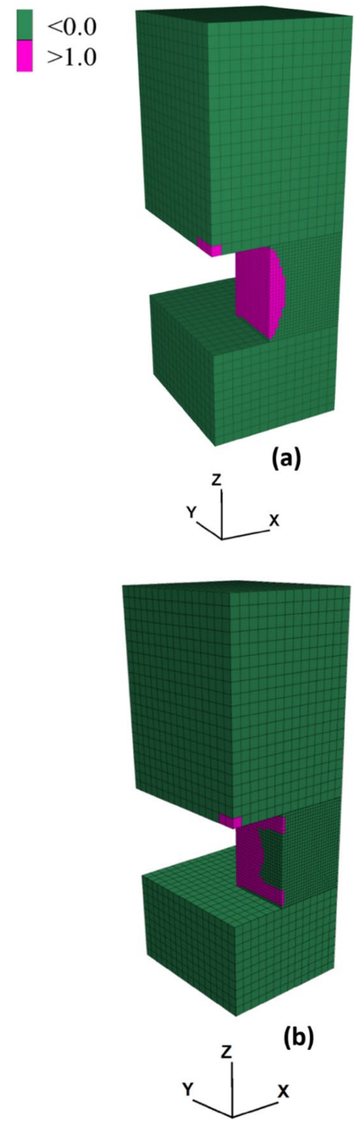

Fig. 6 Regions of strength exceedance zones for **a** classical HB-criterion and **b** extended HB-criterion with b-value of 0.464

more realistic assessment of rock and rock mass strength and allows consequently a more economic mine design as presented exemplary for the pillar design. Depending on rock type and stress situation this effect can be quite significant. Therefore, recognizing the great practical importance, incorporation of the intermediate principal stress component for pillar design and safety considerations is highly recommended.

The authors acknowledge that direct application of laboratory results (parameters) for in-situ dimensioning might not be appropriate due to the scale effect, Nevertheless the stabilizing effect of the intermediate principal stress component remains effective in an in-situ scenario.

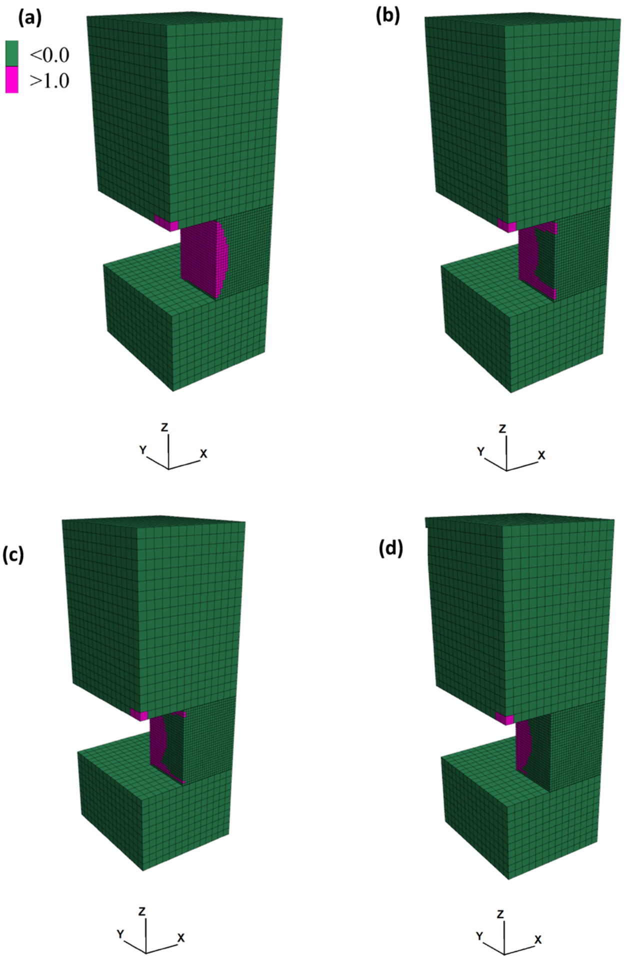

Fig. 7 Effect of b-values on extent of strength violation area for **a** b = 0, **b** b = 0.5, **c** b = 0.7 and **d** b = 1.0

Table 2 Strength exceedance with different b-values

b-value	Total volume of zones with violation of strength criterion (m^3)	Percentage change of total volume with increasing b-values (%)
0 (corresponds to classical HB-criterion)	24.21	100
0.5	12.01	50
0.7	10.44	43
1.0	9.00	37

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Author contributions DM: Conceptualization, Data processing, Drafted the manuscript &Revision. HK: Conceptualization, Review & Editing, Supervision. All authors approved the final manuscript for publication.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. This paper has not been published elsewhere and is not under consideration by another journal.

References

Chen H, Zhu H, Zhang L (2023) Rock slope stability analysis incorporating the effects of intermediate principal stress. Rock Mech Rock Eng. https://doi.org/10.1007/s00603-023-03277-4

Esterhuizen GS (2007) An evaluation of the strength of slender pillars. Trans Min Metall Explor Inc 320:69

Feng XT, Zhang X, Kong R, Wang G (2016) A novel mogi type true triaxial testing apparatus and its use to obtain complete stress-strain curves of hard rocks. Rock Mech Rock Eng 49:1649–1662. https://doi.org/10.1007/s00603-015-0875-y

Feng X-T, Kong R, Zhang X, Yang C (2019) Experimental study of failure differences in hard rock under true triaxial compression. Rock Mech Rock Eng 52(7):2109–2122. https://doi.org/10.1007/s00603-018-1700-1

Feng XT, Kong R, Yang C et al (2020) A three-dimensional failure criterion for hard rocks under true triaxial compression. Rock Mech Rock Eng 53:103–111. https://doi.org/10.1007/s00603-019-01903-8

Haimson B, Chang C (2000) A new true triaxial cell for testing mechanical properties of rock, and its use to determine rock strength and deformability of Westerly granite. Int J Rock Mech Min Sci 37:285–296. https://doi.org/10.1016/S1365-1609(99)00106-9

Hedley DGF, Grant F (1972) Stope-and-pillar design for the Elliot lake uranium mines. Bull Can Inst Min Metall 65:37–44

Hoek E, Brown ET (2019) The Hoek-Brown failure criterion and GSI –2018 edition. J Rock Mech Geotech Eng 11:445–463. https://doi.org/10.1016/j.jrmge.2018.08.001

Hoek E, Carranza C, Corkum B (2002) Hoek-brown failure criterion – 2002 edition. In: Proc.NARMS-TAC Conference. Toronto, pp 267–273

Hu L, Yu L, Ju M, Li X (2022) Journal of rock mechanics and geotechnical engineering effects of intermediate stress on deep rock strainbursts under true triaxial stresses. J Rock Mech Geotech Eng. https://doi.org/10.1016/j.jrmge.2022.06.008

Ingraham MD, Issen KA, Holcomb DJ (2013) Response of Castlegate sandstone to true triaxial states of stress. J Geophys Res Solid Earth 118:536–552. https://doi.org/10.1002/jgrb.50084

Jiang H, Zhao J (2015) A simple three-dimensional failure criterion for rocks based on the hoek-brown criterion. Rock Mech Rock Eng 48:1807–1819. https://doi.org/10.1007/s00603-014-0691-9

Kidega R, Ondiaka MN, Maina D et al (2022) Decision based uncertainty model to predict rockburst in underground engineering structures using gradient boosting algorithms. Geomech Eng 30:259–272

Von Kimmelmann MR, Hyde B, Madgwick RJ (1984) Use of computer applications At Bcl limited in planning pillar extraction and the design of mining layouts. 53–63. https://doi.org/10.1016/0148-9062(85)92716-0

Krauland N, Soder PE (1987) Determining pillar strength from pillar failure observations. Eng Min J 8:34–40

Kwaśniewski M (2013) Recent advances in studies of the strength of rocks under true triaxial compression conditions. Arch Min Sci 58:1177–1200. https://doi.org/10.2478/amsc-2013-0080

Li H, Guo T, Nan Y, Han B (2021) A simplified three-dimensional extension of Hoek-Brown strength criterion. J Rock Mech Geotech Eng 13:568–578. https://doi.org/10.1016/j.jrmge.2020.10.004

Lunder PJ, Pakalnis R (1997) Determination of the strength of hard-rock mine pillars. Bull Can Inst Min Metall 90:51–55

Maleki H (2017) Coal pillar mechanics of violent failure in U.S. Mines Int J Min Sci Technol 27:387–392. https://doi.org/10.1016/j.ijmst.2017.03.001

Martin CD, Maybee WG (2000) The strength of hard-rock pillars. Int J Rock Mech Min Sci 37:1239–1246. https://doi.org/10.1016/S1365-1609(00)00032-0

Mogi K (1971) Fracture and Flow of Rocks under High Triaxial Compression. J Geophys Res 76:1255–1269

Mogi K (1972) Effect of the triaxial stress system on fracture and flow of rocks. Phys Earth Planet Inter 5:318–324. https://doi.org/10.1016/0031-9201(72)90102-1

Potvin Y, Hudyma M, Miller HDS (1990) Rib pillar design in open stope mining. Int J Rock Mech Min Sci Geomech Abstr 27:57. https://doi.org/10.1016/0148-9062(90)90467-g

Sheorey PR, Das MN, Bordia SK, Singh B (1986) Pillar strength approaches based on a new failure criterion for coal seams. Int J Min Geol Eng 4:273–290. https://doi.org/10.1007/BF01552957

Sjoberg J (1992) Failure modes and pillar behaviour in the Zinkgruvan mine. In: proceedings of 33rd U.S. rock mechanics symposium, rotterdam: A.A. Balkema,. Tillerson, J.A., Wawersik, W.R. (Eds.), pp 491–500.

Wagner H (2003) The role of pillars in small underground mines. Int Conf Saf Environ Asp Mining, 63:

Walton G (2021) A New Perspective on the Brittle-Ductile Transition of Rocks. Rock Mech Rock Eng. https://doi.org/10.1007/s00603-021-02595-9

Zhang Q, Zhu H, Zhang L (2013) Modification of a generalized three-dimensional Hoek-Brown strength criterion. Int J Rock Mech Min Sci 59:80–96. https://doi.org/10.1016/j.ijrmms.2012.12.009